EFFECT OF CORONA DISCHARGE ON HEAT TRANSFER IN NATURAL AIR CONVECTION

K. N. Semenov, F. P. Grosu, and M. K. Bologa

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Results are presented of an experimental and theoretical investigation of the effect of a corona discharge on heat transfer from a thin wire under natural air convection conditions.

Despite the fact that the first investigations of electrical convection [1] were made a comparatively long time ago, the application of electrical convection to heat transfer processes has not as yet been studied adequately. The majority of available results on increase of heat transfer under conditions of nonuniform electrical fields in various gases fall in the subdischarge voltage range, and are mainly due to the effect of ponderomotive forces [2]. Only a few papers have dealt with the effect of electrical fields on heat transfer in air, and, since they are directly related to the investigation discussed here, we shall give the main results from them.

In one of the first Russian papers [3], Ostroimov put forward the hypothesis that a change in heat transfer was possible in a homogeneous electric field because of the effect of ponderomotive forces, the hypothesis indicating that hot air must be ejected from the region of maximum field nonuniformity. Using a semishadow method, he established experimentally that a transition from gravitational–thermal convection occurs at a definite field strength, the value required for the appearance of electrical convection being larger, the larger is the temperature of the heat-generating surface. However, it was established later [4] that deformation of the jet of hot air leads to its being directed towards the high-voltage electrode, as if drawn into the strongly nonuniform field. Deformation of the jet is accompanied by the appearance of an electric field, and begins for field strength values larger than those necessary to generate a corona discharge. From this the conclusion was reached that the effect of nonuniform electric fields on heat transfer between air and a thin wire is associated with generation of a discharge.

References [5, 6] give an analysis of convective heat transfer in a nonuniform electric field, on the basis of similarity theory, and give results of an experimental investigation of heat transfer under conditions of natural convection of air. On the basis of a thermal–electric concept, the authors obtained the parameter $K = \Delta\varkappa \,\mathrm{grad}\, E^2 \gamma r^3/\eta^2$, which agrees with the Kronig [7] parameter, to within a constant. However, the experiments showed that the heat transfer increase was negligibly small without a corona discharge: in a constant field it was 5-6%, and even less in an alternating field. But, in the presence of corona, the ratio Q_E/Q_0 reached a value of 2.3 and the experimental correlations for $Q_E/Q_0 = f(U, \Theta_0)$ had maxima both for free and for forced motion of air.

Despite the fact that the value and location of the maximum permit a determination of the effect of the field on the convective heat transfer and a choice of the most favorable conditions for applying this effect, the explanation has not been verified in the literature.

Reference [8] has presented results of an investigation of the effect of a corona discharge on steady and unsteady convective heat transfer in air and in various gases. A correlation was presented for the Nusselt number due to electrical conduction convection, in the form $\mathrm{Nu} = \mathrm{const}\,\mathrm{Gr}\,\mathrm{Pr}\; v/v_T$, where v_T is the speed of the gravitational–thermal wind. This correlation is claimed by the author to be in satisfactory agreement with experimental data only for small currents ($I \leq 60\ \mu A$), since heating of the gas by the corona is not taken into account in this.

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The experimental investigations of [9] have confirmed the presence of a maximum in the dependence of relative heat transfer coefficient on field strength, over a wide range of temperature drop and of high-voltage frequency, and, in addition, a decrease of heat flux was obtained, which features required further experimental and theoretical justification. For this reason the present paper contains an experimental investigation of the convective heat transfer from a thin wire under the action of a corona discharge, in conditions of natural convection of air as a function of the supply voltage and the temperature of the heat-generating surface; in addition, it is proposed to attempt theoretical calculation of heat transfer as a function of corona discharge current, and to compare theoretical and experimental results.

The experimental model is a horizontal copper cylinder of internal diameter D = 17 mm and length l = 120 mm, along the axis of which is stretched a wire of electrolytic nickel of diameter d_0 = 0.2 mm. A constant high voltage of negative polarity was applied to the cylinder in all the tests. The heat-generating wire, heated by a constant current, and simultaneously acting as a resistance thermometer, was connected in a measuring circuit (a double bridge) and was grounded through a microammeter.

Figure 1 shows test results in a study of relative heat transfer with different values of temperature drop. Figure 1a shows relative heat transfer coefficient as a function of potential between the cylinder and the wire, and Fig. 1b shows the corresponding volt–ampere characteristics of the corona discharge. The results obtained (Fig. 1a) are in quite good agreement, both qualitatively and quantitatively, with the data described earlier [9], which confirms that their reproducibility is satisfactory.

The relative heat transfer coefficient was calculated from the ratio $\alpha_E/\alpha_0 = I_E^2/I_0^2$, which follows from the equation for thermal balance of the wire, disregarding end losses.

For all values of the temperature head investigated, the start of a change in wire heat transfer corresponded to the onset of the corona discharge as the voltage on the cylinder was increased. The start of the corona at higher temperatures occurred for smaller field strengths, which is in good agreement with the Peek formula [10], which gives the critical voltage for corona discharge in air at atmospheric conditions.

The shape of the relation $\alpha_E/\alpha_0 = f(U, \Theta_0)$ is determined by the phenomena occurring mainly in the corona layer. Experiments have shown that as long as the power of the corona currents is less than that of the wire heating, the electric field has an intensifying effect on the heat transfer. This is due to increase of mixing of macrovolumes of air by the electric wind, which also leads to an increase in convective heat transfer. When the corona power becomes equal to the thermal power of the wire, the curve reaches a maximum, and for $\Theta_K > Q_E$ it is directed into the region of smaller values of α_E/α_0, again passes through the value of $\alpha_E/\alpha_0 = 1$, reaches zero for specific voltage values increasing with increase of Θ_0. This heat transfer behavior must be a consequence of Joule heating, and experiments have confirmed this conclusively.

It can be seen from Fig. 2 that the relative heat transfer coefficient (curve 1) and the wire temperature for fixed initial temperature head $\Theta_0 = 25°$ (curve 2), as a function of the potential difference, are almost symmetrical relative to their initial values (1.0). This shows that the wire is strongly cooled by the wind up to a voltage corresponding to the greatest value of α_E/α_0, while for larger U, on the contrary, the wire is heated by the corona discharge. The dependence of the wire temperature variation, due only to the corona, on the potential difference between the wire and the cylinder confirms (curve 4) that the wire temperature increase is proportional to the increase in the discharge current I, which is also evidence as to the decisive role of the heat-generation from the corona discharge in diminishing the convective heat transfer. When the last curve was taken the wire carried such a low current that its initial temperature was practically equal to that of the surrounding air layers.

It was noted above that one very probable cause for enhancement of heat transfer in a corona discharge is the electric wind. It is thought that the latter may be accompanied by thermal–electric convection due to nonuniformity of heating of the medium ($\Delta\varkappa$) and to nonuniformity in the electric field (grad E^2). However, in a corona discharge, in the vicinity of the corona layer, where the thermal–electric convection is greatest, the electric field strength, which means also grad E^2, does not depend on the applied voltage [10], and the heat transfer remains at the same level as before the onset of corona (of the order of 5%). Thus, the sharp enhancement of heat transfer on excitation of the corona discharge must be ascribed only to the electric wind. For voltages much greater than the corona ignition threshold, heating by the discharge currents prevails over cooling by the electric wind, which is heated by the corona current. For a

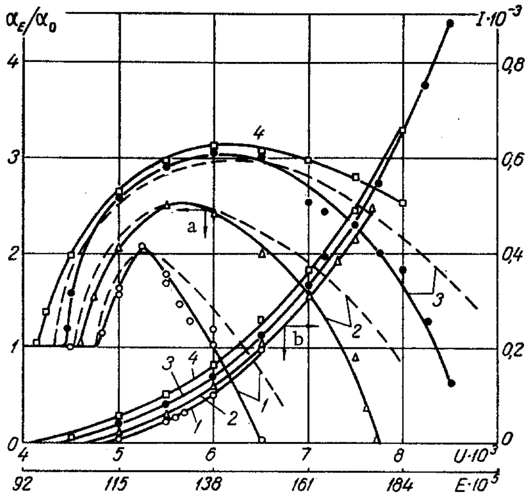

Fig. 1. The relative heat transfer coefficient α_E/α_0 and the corona discharge current I (10^{-3} A) (a and b, respectively) as a function of the potential difference U (10^3 V) between the wire and the cylinder; 1, 2, 3, 4) for temperature drops of 10, 25, 50, and 75° and t_m = 21°C; the solid lines indicate experimental data, and the broken lines indicate calculations according to Eq. (18).

constant temperature of the heat-generating surface, the heat transfer is determined by the ratio between the cooling effect of the electric wind and the heating of the air by the discharge current.

A theoretical solution of the problem of heat transfer with a corona discharge entails integration of a system of nonlinear differential equations in partial derivatives, containing the equation of motion of the medium and the electromagnetic and temperature fields, with appropriate boundary conditions. However, it is difficult to assign boundary conditions with the presently incomplete information concerning the kinematics of the electric wind, which also determines the temperature field. Nevertheless, it is possible to make a quantitative estimate of the phenomena taking place by means of dimensional analysis. The desired quantity in our case is the additional average specific heat flux q' from the surface of the heated wire, due only to the presence of the corona discharge:

$$q' = \lambda \, |\overline{\nabla \Theta}\,|_0. \tag{1}$$

The subscript 0 means that the temperature gradient is determined at the higher surface. Under steady conditions the distribution of temperature Θ satisfies the equation

$$(\mathbf{v}\nabla)\,\Theta = a\,\Delta\Theta + jE/c_p\gamma \tag{2}$$

with boundary conditions $\Theta(r_0) = \Theta_0$, $\Theta(R) = 0$, where v is the velocity of the electric wind.

We shall represent the solution of this linear equation in the form

$$\Theta = \Theta_1 + t_i. \tag{3}$$

Substituting Eq. (3) into Eq. (2) and imposing the condition that the function Θ_1 should satisfy the equation

$$(\mathbf{v}\nabla)\,\Theta_1 = a\,\Delta\Theta_1 \tag{4}$$

with boundary conditions $\Theta_1(r_0) = \Theta_0$ and $\Theta_1(R) = 0$, we obtain an equation for t_i

$$(\mathbf{v}\nabla)\,t_i = a\,\Delta\,t_i + jE/c_p\gamma \tag{5}$$

with the boundary conditions $t_i(r_0) = t_i(R) = 0$.

The result is that the function Θ_1 has the meaning of a temperature distribution when the electric wind is present and no account is taken of Joule energy dissipation with a temperature head equal to Θ_0, and t_i is an increment in the air temperature due to heating by the corona current. Substituting Eq. (3) into Eq. (1), and taking into account that $\nabla t_i|_{r=r_0}$ has the opposite direction to $\nabla\Theta_1$, we obtain

$$q' = \lambda\,|\overline{\nabla\Theta_1}\,|_0 - \lambda\,|\,\overline{\nabla t_i}\,|_0. \tag{6}$$

The first term of this equality can be evaluated by omitting integration of Eq. (4). Since the length of wire in the corona is much greater than its diameter, it is reasonable to assume that the transverse flow of the electric wind over the wire will be two-dimensional. Because of the small wire radius it can be argued that the boundary layer due to this flow is laminar. For these assumptions and for values of Re and Pr not too small, we have the relation

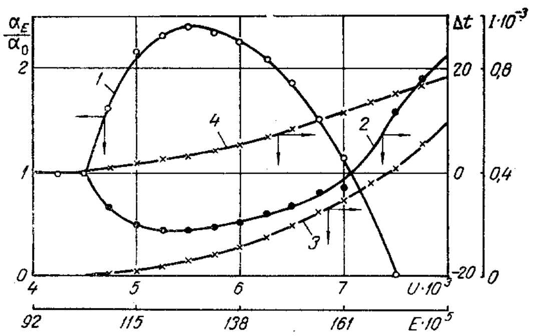

Fig. 2. The relative heat transfer coefficient α_E/α_0 (1), the variation in wire temperature Δt (deg) for $\Theta_0 = 25°$ (2), the corona current I (10^{-3} A) (3), and the temperature increment in the corona wire Δt (deg) for $\Theta_0 = 0$ (4) as a function of the potential difference U (10^3 V) between the wire and the cylinder.

$$\mathrm{Nu} = \mathrm{const}\,\mathrm{Pr}^{1/3}\,\mathrm{Re}^{1/2}, \tag{7}$$

where the Re number describes the rate of approach of the medium to the heat-generating surface. For this velocity we take the volume mean of the absolute value of the electric wind velocity, which we determine from dimensional considerations. It follows from the Navier–Stokes equation, containing the Coulomb force $\rho E = i/2\pi rk$, causing the electric wind, that the velocity is determined by the following parameters: i/k, γ, η. Because the process which interests us occurs in the immediate vicinity of the heat-generating surface, it is natural to choose the characteristic dimension to be d_0, but, on the other hand, the hydrodynamic phenomena also depend strongly on the distance between the wire and the outer cylinder, and therefore we expect that the velocity will depend additionally on the ratio D/d_0. Taking the above into account we find $v = v(i/k, \gamma, \eta, d_0, D/d_0)$. It transpires that we can establish the specific form of this function comparatively simply by examining two limiting cases:

1) for Re $\ll 1$ $v = v_1(i/k, \eta, d_0, D/d_0)$,

2) for Re $\gg 1$ $v = v_2(i/k, \gamma, d_0, D/d_0)$.

In the first case the unique combination of the dimensional quantities enclosed in brackets above which will have the dimension of velocity is

$$v_1 = \frac{id_0}{k\eta}\varepsilon_1, \tag{8a}$$

and, in the second case,

$$v_2 = \left(\frac{i}{k\gamma}\right)^{1/2}\varepsilon_2. \tag{8b}$$

Here ε_1 and ε_2 are dimensionless coefficients of proportionality, depending on the ratio D/d_0: for $D/d_0 \to 1$ ε_1 and $\varepsilon_1 \to 0$, and for $D/d_0 \to \infty$, these tend to numerical constants.

The limiting cases of Eqs. (8a) and (8b) follow from the expression

$$\bar{v} = \frac{\eta}{\gamma d_0}\left(\frac{id_0^2\gamma}{k\eta^2}\right)^m \varepsilon(D/d_0) \tag{9}$$

with m = 1 and m = 0.5, respectively, and the Reynolds number given by Eq. (9) is

$$\mathrm{Re} = \varepsilon\left(\frac{id_0^2}{k\gamma\nu^2}\right)^m. \tag{10}$$

Substituting the latter into Eq. (7), we obtain

$$\mathrm{Nu}_1 = \mathrm{const}\,\varepsilon^{1/2}\,\mathrm{Pr}^{1/3}\left(\frac{id_0^2}{k\gamma\nu^2}\right)^{m/2}. \tag{11}$$

From Eq. (11) we find the specific heat flux determining the first term of Eq. (6):

$$\lambda\,|\overline{\nabla\Theta_1}|_0 = \mathrm{Nu}_1\frac{\lambda\Theta_0}{d_0}$$

or, allowing for Eq. (11)

$$\lambda |\overline{\nabla\Theta_1}|_0 = n\,\mathrm{Pr}^{1/3}\,\frac{\lambda\Theta_0}{d_0}\left(\frac{Id_0^2}{lk\,\gamma\nu^2}\right)^{m/2} = AI^{m/2}, \tag{12}$$

where n is a dimensionless coefficient depending on the ratio D/d_0.

For small corona discharge currents, when Joule heating can be neglected, this expression gives the effect of the electric field on the heat transfer.

For large discharge currents we must take into account Joule heating of the air. To do this we have to solve Eq. (5), which differs from the original Eq. (2), only in the boundary conditions. However, the temperature increment in Eq. (5) is greatest in the corona layer close to the wire surface, which is quite thin compared to the size of the discharge gap. In the outer region of the corona discharge Joule heating can be neglected in the zero-order approximation. Therefore, to estimate the second term of Eq. (6) we can neglect the convective term in Eq. (5): in the corona layer, because v is small, and in the external region, because all its components are small.

In this approximation Eq. (5) can be written as follows:

$$\begin{gathered}\operatorname{div}\mathbf{q} + \mathbf{j}\cdot\mathbf{E} = 0,\\ \mathbf{q} = -\lambda\,\mathrm{grad}\,t_i.\end{gathered} \tag{13}$$

We have the following approximations for the electric current density and the field strength, in accordance with [10]:

$$j = \frac{I}{2\pi rl}, \tag{14a}$$

$$E = \frac{r_0 E_\text{к}}{r} \quad \text{for} \quad r_0 \leqslant r \leqslant r_i, \quad r_i = \frac{E_\text{к} r_0}{E_i}, \tag{14b}$$

$$E = \sqrt{\frac{2I}{kl}} \quad \text{for} \quad r_i \leqslant r \leqslant R. \tag{14c}$$

Equation (14c) is applicable for large discharge currents. However, to determine q_{r_0} we can use it even for small current strengths, since then we can generally neglect heating in the outer corona region.

Following a first integration of Eq. (13), in cylindrical coordinates, we have:

$$rq = \begin{cases} r_0 q_{r_0} - \dfrac{I}{2\pi l}\displaystyle\int_{r_0}^{r} E\,dr, & r_0 \leqslant r \leqslant r_i, \quad (15a)\\[2ex] r_i q_{r_i} - \dfrac{I}{2\pi l}\displaystyle\int_{r_i}^{r} E\,dr, & r_i \leqslant r \leqslant R. \quad (15b)\end{cases}$$

Integrating Eqs. (15a) and (15b) within the end limits (from r_0 to r_i and from r_i to R), and allowing for the boundary conditions for t_i, we obtain

$$-\lambda\, t_{r_i} = \begin{cases} r_0 q_{r_0}\ln\dfrac{r_i}{r_0} - \dfrac{I}{2\pi l}\displaystyle\int_{r_0}^{r_i}\frac{dr}{r}\int_{r_0}^{r} E\,dr, & r_0 \leqslant r \leqslant r_i, \quad (16a)\\[2ex] -r_i q_{r_i}\ln\dfrac{R}{r_i} + \dfrac{I}{2\pi l}\displaystyle\int_{r_i}^{R}\frac{dr}{r}\int_{r_i}^{r} E\,dr, & r_i \leqslant r \leqslant R. \quad (16b)\end{cases}$$

On the other hand, from Eq. (16a) we find

$$r_i q_{r_i} = r_0 q_{r_0} - \frac{IE_\text{к} r_0}{2\pi l}\ln\frac{r_i}{r_0}.$$

Substituting $r_i q_{r_i}$ into Eq. (16b) and subtracting Eq. (16b) from Eq. (16a), following integration over the above limits, we obtain

$$q_{r_0} = \frac{IE_{\text{к}}}{2\pi l \ln \frac{R}{r_0}} \left[\ln \frac{E_{\text{к}}}{E_i} \ln \left(\frac{R}{r_0} \sqrt{\frac{E_i}{E_{\text{к}}}} \right) + \frac{\sqrt{\frac{2I}{kl}}}{E_i} \left(\frac{RE_i}{r_0 E_{\text{к}}} - \ln \frac{eRE_i}{r_0 E_{\text{к}}} \right) \right] = BI. \tag{17}$$

Substituting Eqs. (12) and (17), which yield the coefficients A and B, into Eq. (6), we find

$$q' = AI^{m/2} - BI.$$

The total specific heat flux q_E can be determined as the sum of q' and q_0:

$$q_E = AI^{m/2} - BI + q_0.$$

By dividing the right and left side by q_0, we obtain the following relation for the relative heat transfer coefficient:

$$\frac{\alpha_E}{\alpha_0} = \frac{1}{q_0} (AI^{m/2} - BI) + 1. \tag{18}$$

To determine m and n, we write Eq. (18) in the form

$$\frac{m}{2} x + b = y,$$

where

$$x = \ln \left(\frac{Id_0^2}{lk\gamma\nu^2} \right), \quad b = \ln n, \quad y = \ln \frac{[\alpha_E/\alpha_0 + BI/q_0 - 1]\, d_0 q_0}{\lambda \Theta_0 \sqrt[3]{\Pr}}.$$

In this interpretation the experimental data shown in Fig. 1 are described by a line which has a kink at x = 7 (which corresponds to current strengths of about 10^2 μA), from which m and n can be approximated as follows:

$$n = \begin{cases} 0.105 & \text{for} \quad I \leqslant 100 \ \mu\text{A}, \\ 0.256 & \text{for} \quad I \geqslant 100 \ \mu\text{A}; \end{cases}$$

$$m = \begin{cases} 0.89 & \text{for} \quad I \leqslant 100 \ \mu\text{A}, \\ 0.66 & \text{for} \quad I \geqslant 100 \ \mu\text{A}. \end{cases}$$

Figure 1 shows curves calculated from Eq. (18), in addition to the experimental relations for the heat transfer coefficients. Comparison shows that the discrepancy between experiment and theory is no more than 10%. We can therefore conclude that Eq. (18) in the main correctly represents the nature of the effect of corona discharge on heat transfer under natural air convection conditions.

NOTATION

r_0, d_0	are the radius and diameter of heat-generating wire;
R, D	are the radius and diameter of outer cylinder;
r_i	is the radius of corona layer;
r	is the ambient radius;
l	is the operating length of wire;
E	is the electric field strength;
E_K	is the initial corona voltage;
E_i	is the voltage at which ionization begins due to collisions of the first kind;
Q_E, q_E, α_E, I_E	are the heat flux, specific heat flux, heat transfer coefficient, wire heating current with an electric field present;
Q_0, q_0, α_0, I_0	are the same quantities with E = 0;
Q_K, q	are the heat flux and specific heat flux due to corona discharge;
q_{r_0} and q_{r_i}	are the same at the wire surface and at distance r_i from the wire axis;
I	is the corona current;
U	is the potential difference between wire and cylinder;
k	is the mobility of ions of the sign of the corona electrode;
i	is the corona current per unit wire length;
Pr	is Prandtl number;
Re	is Reynolds number;

Θ_0 is the temperature drop;
t is the wire temperature;
t_m is the medium temperature;
Δt is the wire temperature change due to corona discharge current;
t_i is the temperature of medium due to heating by discharge current;
t_{r_i} is the same at distance r_i from the wire axis;
$\bar{v}$ is the mean velocity of electric wind;
ρ is the discharge density in the outer corona region;
$\varkappa$ is the polarization coefficient;
λ is the thermal conductivity;
ν is the kinematic viscosity;
η is the dynamic viscosity;
γ is the air density;
e is the base of natural logarithms;
c_p is the specific heat at constant pressure.

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